INDEX

General Aptitude

NUMBER SYSTEM

CHAPTER 1

-: GENERAL APTITUDE :-

A number is an arithmetical value, expressed by a symbol or word, representing a particular quantity or amount and used in counting and making calculations.

1.1 Different Types of Numbers:

1.1.1 Natural Numbers

The counting numbers are called natural numbers. Natural numbers do not include 0 or any negative numbers.

Example: 1, 2, 3, 4… are all-natural numbers.

1.1.2 Whole Numbers

All counting numbers and 0 (zero) together form the set of whole numbers. They do not include negative numbers.

Example: 0, 1, 2, 3, 4… are whole numbers.

1.1.3 Integers

All counting numbers including 0 and the negatives of the counting numbers form the set of integers.

Example: -3, -2, -1, 0, 1, 2, 3… are all integers.

Set of positive integers = {1, 2, 3, 4, 5, 6…}

Set of negative integers = {-1, -2, -3, -4, -5, -6…}

Set of non-negative integers = {0, 1, 2, 3, 4, 5, 6…}

1.1.4 Even Numbers and Odd Numbers

An integer which is exactly divisible by 2 is called an even number. While an integer which cannot be exactly divisible by 2 is called an odd number.

Example: -6, -4, -2, 0, 2, 4, 6, 8, 10… are all **even numbers**.

Zero is considered an even number.

While an integer which cannot be exactly divisible by 2 is called an odd number.

Example: -5, -3, -1, 1, 3, 5, 7, 9, 11… are all **odd numbers**.

1.1.5 Prime Numbers

A natural number other than 1 is a **prime number** if it is divisible by 1 and itself only.

Example: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43 and 47 are the prime numbers less than 50.

Test to find whether a given number is a prime:

Step 1:

Select a least positive integer n such that $n^2 >$ given number

Step 2:

Test the divisibility of a given number by every prime number less than n.

Step 3:

The given number is prime only if it is not divisible by any of these primes.

Example: Check whether 571 is a prime number

Since $(23)^2 = 529 < 571$ and $(24)^2 = 576 > 571$

So, n = 24

Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23.

Since 571 is not divisible by any of these primes, therefore 571 is a prime number.

Example: Check whether 923 is a prime number.

Since $30^2 = 900 < 923$ and $31^2 = 961 > 923$

So, n = 31

Prime numbers less than 31 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Since 923 is divisible by 13, 923 is not a prime number

1.1.6 Composite Numbers

Natural numbers greater than 1 which are not prime are known as **composite numbers**.

Example: 9 can be divided by 3 (as well as 1 and 9), so 9 is a composite number similarly 12 can be divided by 2, 3, 4 and 6, so it is also a composite number.

Note:

1. The number 1 is neither a prime number nor a composite number.
2. The number 2 is the only even number which is prime

1.1.7 Rational Numbers

A rational number is a number that can be represented as a ratio or fraction in which both the numerator and the denominator are whole numbers.

The number 18 can be said to a rational number because it can be written as the fraction $\frac{18}{1}$.

Likewise, ¾ is also a rational number because it can be written as a fraction.

Every whole number is a rational number, because any whole number can be written as a fraction.

1.1.8 Irrational Numbers

Any number that is not rational is considered as an irrational number. An irrational number can be written as a decimal, but not as a fraction.

An irrational number has endless non-repeating digits to the right of the decimal point.

Some important examples of irrational numbers include:

π = 3.141592…

$\sqrt{2}$ = 1.414213…

Although irrational numbers are not often used in daily life, they do exist on the number line. In fact, between 0 and 1 on the number line, there are an infinite number of irrational numbers

1.1.9 Decimal Numbers

Not every number we face is a whole number. Our decimal system allows us a flexibility in writing the numbers of different values and sizes, using a symbol called the decimal point.

Example: 32.567, 12.87 23.64

As we move right from the decimal point, each place value is divided by 10. While on moving to the left side from the decimal point each place value is multiplied by 10.

1.1.10 Perfect Number:

A perfect number is a natural number, which when you add up all of the factors less than that number will give you that number.

Example: The factors of 28 are 1, 2, 4, 7, and 14.

1 + 2 + 4 + 7 + 14 = 28.

Thus, 28 is a perfect number.

1.1.11 Perfect Square:

A perfect square is a number that can be written as the product of two equal factors.

Example: 25, 36, 49, 64… are all perfect squares.

Important properties of perfect squares:

(a) No perfect square ends with 2, 3, 7, 8.

(b) No perfect square ends with an odd number of zeros.

(c) Perfect squares can be recognized by the fact that all of their prime factors have even multiplicities.

1.1.12 Fractions:

A fraction is used to represent a part of a whole or any number of equal parts. A fraction (examples: 5/11 and 3/7) consists of an integer numerator, displayed above a line, and a non-zero integer denominator, displayed below that line.

Proper Fraction:

A fraction whose numerator is less than the denominator. The value of such fractions is usually less than 1.

Example $\frac{1}{4}, \frac{2}{7}, \frac{17}{20}$.

Improper Fraction:

A fraction whose numerator is equal to or greater than the denominator.

The value of such fractions is usually more than 1.

Example. $\frac{5}{3}, \frac{10}{7}, \frac{40}{19}$.

Mixed Fraction:

An improper fraction (fractions having denominator smaller than numerator) can be expressed as a whole number and a proper fraction. Such fractions are then called mixed fraction.

Steps involved in conversion of improper fraction to mixed fraction

Step 1: Divide the numerator by the denominator.

Step 2: Write down the Quotient (whole number) of the division followed by the fraction having the remainder above the denominator.

Example $5\frac{1}{4}, 3\frac{7}{8}, 22\frac{3}{17}$.

Factorization:

The prime numbers that divide an integer exactly are termed as the prime factors of that Integer.

The factorization of a positive integer is the process of listing of the integer's prime factors, together with their multiplicities.

Example 1: Prime factors of 420 $= 2 \times 2 \times 3 \times 5 \times 7$

$= 2^2 \times 3^1 \times 5^1 \times 7^1$

Where, 2, 3, 5 and 7 are factors having multiplicities 2, 1, 1 and 1 respectively.

Example2: Prime factorization of 900 = $2^2 \times 3^2 \times 5^2$ in which multiplicities of the factors 2, 3, 5 are 2, 2, 2 respectively, which are all even. Thus 900 is a perfect square.

IV. Perfect square (unit digit- 0, 1, 4, 5, 6, 9)

Unit's digit	0	1	4	5	6	9
Ten's digit	0	even	even	2	odd	even

1.2 Factors and Multiples:

If a number, say, 'x' exactly divides another number 'y', we can say that 'x' is a factor of 'y'. Further, 'y' is called a multiple of 'x'.

For example, 4 is a common factor of 8 and 12.

Example 1: If A and B are two natural numbers such that A *x* B = 84, which of the following cannot be the value of A + B?

(1) 21 (2) 25

(3) 45 (4) 85

Explanation:
Checking only the given answer options,
12 x 7 = 84, 12+7 = 21
21 x 4 = 84, 21+4 = 25
84 x 1 = 84, 84+1 = 85
42 x 2 = 84, 42+2 = 44
Since A and B are natural numbers and we can get 44 by addition of a particular set of values, there can be no way of getting 45 by using natural number for A and B.
Therefore, 45 is the number which cannot be obtained in the above question i.e. option (3)
Answer (3)

1.2.1 GCD - Greatest Common Divisor (Highest Common Factor (HCF)):
The GCD or HCF of two or more than two numbers is the greatest number that divides each one of them exactly.
For example, 6 is the highest common factor of 12, 18 and 24.

Method for finding the GCD of two or more numbers:
Step 1: Express each one of the given numbers as the product of prime factors.
Step 2: Choose common factors.
Step 3: Find the product of these common factors. This is the required H.C.F. of given numbers.
Example: Find the GCD of 360 and 240?
Prime factors of 360 = 2 × 2 × 2 × 3 × 3 × 5 = $2^3 3^2 5^1$
Prime factors of 240 = 2 × 2 × 2 × 2 × 3 x 5 = $2^4 3^1 5^1$
Prime factors common to 360 and 240= 2 × 2 × 2 × 3 x 5 = 120.
So, the GCD/HCF of 360 and 240 is 120.

1.2.2 Least Common Multiple (LCM):
The LCM of two or more given numbers is the smallest number which is exactly divisible by each one of them.

Method of finding LCM of two or more numbers:
Step 1: Resolve each given number into prime factors.
Step 2: Take out all factors with highest powers that occur in given numbers.
Step 3: Find the product of these factors. This product will be the L.C.M.

Example 1: Find LCM of 360 and 240
Prime factors of 360 = 2 × 2 × 2 × 3 × 3 × 5 = $2^3 3^2 5^1$.
Prime factors of 240 = 2 × 2 × 2 × 2 x 3 × 5 = $2^4 3^1 5^1$.
Product of highest powers of all prime factors of 360 and 240 = 2^4 x 3^2 × 5^1 = 720.
Thus, the LCM of 360 and 240 is 720.

Important Properties of GCD and LCM

i. Product of two numbers = LCM of the numbers × HCF of the numbers
ii. HCF or GCD of given numbers always divides their LCM
iii. HCF or GCD of given fractions = $\frac{GCD\ of\ Numerators}{LCM\ of\ Denominator}$
iv. LCM of given fractions = $\frac{LCM\ of\ Numerators}{GCD\ of\ Denominator}$

Example 2: The GCD of two numbers is 2 and their LCM is 42. If one of the numbers is 6, find the other number?
(1) 12 (2) 13
(3) 14 (4) 16
Explanation:
Let another number be y.
We know that, Product of two numbers = Product of their GCD and LCM
i.e. $6 \times y = 2 \times 42$
i.e. $y = \frac{(2 \times 42)}{6}$
i.e. y = 14
Answer (3)

Example 3: Find the H.C.F of 108, 360 and 600.
(1) 12 (2) 13
(3) 14 (4) 15
Explanation:
Upon prime factorization of the given numbers,
108 = 2^2 x 3^3,
360 = (2^3 x 3^2 x 5)
600 = (2^3 x 5^2 x 3)
Taking out the common factors, we can say,
H.C.F = (2^2 x 3)
= (4 x 3)
= 12
Answer (1)

1.3 'BODMAS' Rule:
This rule depicts the correct sequence in which the operations are to be executed, so as to find out the value of the given expression.
Here B - Bracket,
O - of,
D - Division,
M - Multiplication,
A - Addition and
S – Subtraction
Thus, in simplifying an expression, first of all the brackets must be removed, strictly in the order (), {} and ||.
After removing the brackets, we must use the following operations strictly in the order:
(i) Division (ii) Multiplication (iii) Addition (iv) Subtraction.

Example 4: What will be the value of (48 - 12) ÷ 4 + 6 ÷ 2 × 3?

(1) 18 (2) 9/4

(3) 24 (4) 4.5

Explanation:

(48 - 12) ÷ 4 + 6 ÷ 2 × 3

= 36 ÷ 4 + 6 ÷ 2 × 3 (Solving Bracket)

= 9 + 3 × 3 (Solving Division)

= 9 + 9 (Solving Multiplication)

= 18 (Solving Addition)

Answer (1)

1.4 Test for Divisibility:

- A number is **divisible by 2** if its last digit is 0 or divisible by 2 (i.e. 0,2,4,6 or 8).
  Example – 48, 786, 987654 are divisible by 2
- A number is **divisible by 3** if the sum of its digits is divisible by 3.
  Example: 534: 5+3+4=12 and 1+2=3 so 534 is divisible by 3.
- A number is **divisible by 4** if the last two digits of the number are divisible by 4 or both are 0.
  Example – 4224 is divisible by 4 since 24 (last two digits are divisible by 4)
- A number is **divisible by 5** if the last digit is 5 or 0.
  Example – 555, 8870, 745 are divisible by 5
- A number is **divisible by 6** if the number is divisible by both 2 and 3.
  Example –36, 90, 216 are divisible by 2 since they are even and also by 3 (applying divisibility test for 3), therefore this numbers should be divisible by 6.
- A number is **divisible by 7** if the number follows the given condition –
  Double the last digit and subtract it from the remaining leading truncated number. If the result is divisible by 7, then so was the original number.
  Example – Whether 357 is divisible by 7 or not can be checked by taking the unit digit 7, double it to 14, and then subtract it from 35, then 35 – 14 = 21, which is divisible by 7. Therefore, 357 is divisible by 7.
- A number is **divisible by 8** if the number formed by the last three digits is divisible by 8.
  Example – 376, 67192, 1225448 are divisible by 8.
- A number is **divisible by 9** if the sum of the digits is divisible by 9.
  Example – 459, 1242, 6732 are divisible by since their addition is divisible by 9.
- A number is **divisible by 10** if it ends in a 0.
  Example – 340, 5670, 10000 are divisible by 10.
- A number is **divisible by 11** if the difference of the sum of the digits in the even and odd positions gives us a number which is either 0 or divisible by 11.
  Example – 572 is divisible by 11,
  Sum of digits in the even positions (i.e.2nd position) = 7
  Sum of digits in the odd positions (i.e.1st and 3rd position) = 5 + 2 = 7
  Difference = 7 -7 = 0, which is divisible by 11.
  There is an **alternative method for testing the divisibility by 11**.
  Subtract the last digit from the remaining leading truncated number. If the result is divisible by 11, then so was the first number. Apply this rule over and over again as necessary.
  Example – 19151→ 1915-1 =1914 →191-4=187 →18-7=11,
  so yes, 19151 is divisible by 11
- A number is divisible by 12 if the number is divisible by 2, 3 and 4 as well.
  Example – 108 is divisible by 2(since its even), divisible by 3(since addition of the digits is 9), divisible by 4(since 08 is divisible by 4) and therefore divisible by 12.
- A number is divisible by 13 if it follows the following condition- Add four times the last digit to the remaining leading truncated number. If the result is divisible by 13, then so was the first number. Apply this rule over and over again as necessary.
  Example–50661-->5066+4=5070-->507+0=507--> 50+28=78 and 78 is 6×13, so 50661 is divisible by 13.

Example 5: The number 333,333,333,333 is divisible by

(1) 9 and 11 (2) 3 and 11

(3) 3 and 9 (4) 3, 9 and 11

Explanation:

The addition of the digits in the number is 36 which is divisible by 3 as well as 9 therefore the number must be divisible by both 3 and 9.

Let us check the divisibility test for 11,

So, the addition of the digits at even places will be

3 +3 +3 +3 +3 +3 = 18,

While the addition of the digits at odd places will be 3 +3 +3 +3 +3 +3 = 18

So, 18-18 = 0, therefore according to the divisibility test of 11, the number should be divisible by 11 as well.

Answer (4)

1.5 Modulus of a Real Number:

Modulus of a real number x is defined as:

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

Thus, |5| = 5 and |-5| = -(-5) = 5.

1.6 Laws of Exponents:

X^a:When 'X' is a natural number, then X^astands for the product of 'a' factors each equal to 'X'.

- $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$
- $x^0 = 1$
- $x^{-a} = \frac{1}{x^a}$
- $x^{\frac{1}{a}} = \sqrt[a]{x}$
- For any natural numbers a, b, c, d,
  $a^b > a^c > a^d$, if and only if $b > c > d$.
- For three positive real numbers 'a', 'b', 'c' with the same positive valued exponent x,
  $a^x > b^x > c^x$if and only if $a > b > c$.

1.7 Properties of Exponents:

- $x^{\,a} \times x^{\,b} = x^{\,a+b}$
- $\frac{x^a}{x^b} = x^{\,a-b}$
- $(x^a)^b = x^{\,ab}$
- $(xy)^a = x^a . y^a$.

Example 6

$(32)^{2.6}$ x $(32)^?$ = 32^8

(1) 2.9 (2) 2.75

(3) 4.5 (4) 5.4

Explanation:

$(32)^{2.6}$ x $(32)^?$ = 32^8

$(32)^? = \frac{32^8}{32^{2.6}}$

$(32)^? = 32^{8-2.6}$ (since the numbers (roots) of the power are same)

$(32)^? = 32^{5.4}$

Answer (4)

Some Important Expansion Formulae:

i) $(x + y)(x - y) = (x^2 - y^2)$
ii) $(x + y)^2 = (x^2 + y^2 + 2xy)$
iii) $(x - y)^2 = (x^2 + y^2 - 2xy)$
iv) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$
v) $(x^3 + y^3) = (x + y)(x^2 - xy + y^2)$
vi) $(x^3 - y^3) = (x - y)(x^2 + xy + y^2)$
vii) $(x^3 + y^3 + z^3 - 3xyz) = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$
viii) When $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$.

1.8 Quadratic Equation

A quadratic equation is an equation of the form $f(x) = ax^2 + bx + c$, where a, b, c are all real and a≠0, and x is an unknown variable.

The values of x satisfying $f(x) = 0$ are called its roots.

- In $ax^2 + bx + c = 0$, the expression $D = b^2 - 4ac$ is called discriminant.
- Let α, β be the roots of the equation, $ax^2 + bx + c = 0$. Then
  Sum of roots= $\boldsymbol{\alpha + \beta = \frac{-b}{a}}$
  Product of roots= $\boldsymbol{\alpha\beta = \frac{c}{a}}$
- When the roots of quadratic equation are given, the quadratic equation is given by
  x^2 −(Sum of roots) x + Product of roots= 0

Nature of the roots of $ax^2 + bx + c = 0$

Let $D = b^2 - 4ac$ be the discriminant of the given equation and the nature of two roots depends on this discriminant as follows:

1. Real and equal if $D = 0$
2. Real, unequal and rational, when $D > 0$ and D is a perfect square.
3. Real, unequal and irrational, when $D > 0$ but D is not a perfect square
4. Imaginary, if $D < 0$

Example 7: While solving a problem that reduces to a quadratic equation one student makes a mistake only in the constant term of the equation and obtains 8 and 2 for the roots. Another student makes a mistake only in the coefficient of the first-degree term and finds -9 and -1 for the roots. The correct equation was:

(1) $x^2 - 10x + 9 = 0$ (2) $x^2 + 10x + 9 = 0$

(3) $x^2 - 10x + 16 = 0$ (4) $x^2 - 8x - 9 = 0$

Explanation:

For a general quadratic equation, $px^2 + qx + r = 0$,

Sum of roots = $-\frac{q}{p}$,

Product of roots = $\frac{r}{p}$

Here $x^2 + ax + b = 0$

So, p = 1, q = a, r = b.

Thus, sum of roots = -a

Product of roots = b

First student took the wrong value of b, but the correct value of a. Therefore, the sum of roots is correct.

Thus, sum of the roots (-a) = 8 + 2 = 10

So, -a = 10

a = -10

Another student took a wrong value of a, but correct value of b.

Therefore, the products of roots are correct.

Thus, product of roots (b) = -9 x -1 = 9

b = 9

Thus, the correct values of a and b are -10 and 9

Therefore, the correct equation becomes $x^2 - 10x + 9 = 0$

Answer (1)

1.9 Power Cycle of Number:

When we multiply any two numbers having more than one digit, the last digit of the product is the result of the last digit of the product of the last digit of the two numbers.
For example, 332 × 332 =110224, the last digit of this product can be simply obtained by multiplying 2 × 2 = 4 and taking the last digit of that product, i.e., 4 itself.
The last digit of a number x^y forms an order which depends on the unit (last) digit of the number (x) and the power of the number i.e. (y).
This order which is followed by a particular number is the power cycle of that number.
Example – The power cycle of 3:

3^1 = **3**, 3^5 = 24**3**, 3^9 = 1968**3**,
3^2 = **9**, 3^6 = 72**9**, 3^{10} =5904**9**,
3^3 = 2**7**, 3^7 = 218**7**, 3^{11} = 17714**7**,
3^4 = 8**1**, 3^8 = 656**1**, 3^{12} = 53144**1**

Notice that the unit digit gets repeated after every 4th power of 3.
Therefore, it is said that 3 has a power cycle of 3, 9, 7, 1 with frequency 4.

All the numbers having 3 at their unit place will have the power cycle and its frequency is the same as 3.
The power cycle of the digits from 0-9 is given below:

Last Digit	Power Cycle	Frequency
0	0	1
1	1	1
2	2, 4, 8, 6	4
3	3, 9, 7, 1	4
4	4, 6	2
5	5	1
6	6	1
7	7, 9, 3, 1	4
8	8, 4, 2, 6	4
9	9, 1	2

Power cycle of any given number can be estimated by knowing the power cycle of these 9 digits.

Example 8: What is the last digit of 38^{38}?
(1) 2 (2) 4
(3) 6 (4) 8
Explanation:
The unit place digit of 38 is 8. From the above table we know that power cycle of 8 is {8, 4, 2, 6} with the frequency of 4.
That is, 8^1 = 8,
8^2 = 64,
8^3 = 512,
8^4 = 4096 and so on.
Since the power of 38 is 38, we can directly divide 38 by 4 and get remainder as 2 which means the last digit of the number we get after raising the power to 38 is same as the last digit of power 2 and i.e.4
Answer (2)

Example 9: What is the last digit of 63^{121}?
(1) 1 (2) 3
(3) 7 (4) 9
Explanation:
To obtain the last digit of 63^2, 63^3… multiply the last digits i.e., 3×3, 9×3, 7×3 etc.
Hence, the last digit is given by,
For 63^1, Last digit = 3, i.e.3
For 63^2, Last digit = 3 × 3 = 9, i.e., 9
For 63^3, Last digit = 9 × 3 = 27, i.e., 7
For 63^4, Last digit = 7 × 3 = 21, i.e., 1.
For 63^5, Last digit = 1 × 3 = 3, i.e., 3
For 63^6, Last digit = 3 × 3 = 9, i.e., 9
For 63^7, Last digit = 9 × 3 = 27, i.e., 7
For 63^8, Last digit = 7 × 3 = 21, i.e., 1.
Clearly, we can observe that the unit digit gets repeated after every 4th power of 3.
Hence, we can say that 3 has a power cycle of 3, 9, 7, 1, with frequency 4.
Therefore, for powers, 1, 5, 9, 13… the last digit will be 3.
For powers, 2, 6, 10, 14…., the last digit will be 9.
For powers, 3, 7, 11, 15…., the last digit will be 7.
For powers, 4, 8, 12, 16…., the last digit will be 1.
In the question, the power is 121.
We have to check, in which series does 121 come.
121 is a number that gives a remainder 1 when divided by 4.
Therefore, 121 is in the series 1, 5, 9, 13, …
Hence, the last digit is 3.
Answer (2)

Example 10: When 2^{33} is divided by 10, the remainder will be
(1) 2 (2) 3
(3) 4 (4) 8
Explanation:
We know that, 2^1= 2, 2^2= 4, 2^3= 8, 2^4= 16, 2^5= 32….
2 have a power cycle {2, 4, 8, 6} with the frequency of 4. So, dividing 33 by 4 we get 1 as remainder.
So, 2^1= 2 should be present on unit place.
The number present on unit place is the remainder when any number is divided by 10.
Answer (1)

1.10 Addition of the Sequences

Arithmetic Sequence

In an arithmetic sequence, when each term of a sequence differs from its preceding term by a constant value, the value is called the common difference of that sequence.

The general form of an arithmetic sequence is a, a+d, a+2d, a+3d and so on.

Three numbers x, y and z are in A.P. if $2y = x + z$

The n^{th} term of an Arithmetic Sequence is can be found by
$t_n = a + (n-1)\,d$

Sum of n terms of an Arithmetic sequence is given by
$S_n = \frac{n}{2}$(first term + last term)
$= \frac{n}{2}[2a + (n-1)\,d]$
Where, a = first term of the sequence,
d = common difference of the sequence = $T_n - T_{n-1}$
Sum of first n consecutive natural numbers
$= (1 + 2 + 3 + \ldots + n) = \frac{n\,(n+1)}{2}$
Sum of first n consecutive square numbers
$= (1^2 + 2^2 + 3^2 + \ldots + n^2) = \frac{n(n+1)\,(2n+1)}{6}$
Sum of first n consecutive cubic numbers
$= (1^3 + 2^3 + 3^3 + \ldots + n^3) = \frac{(n\,(n+1))2}{4}$

1.11 Geometric Sequence

In a geometric sequence, the ratio of a term with its preceding term tends to be constant.
The general form of a G.P. is a, ar, ar^2, ar^3 and so on.
Three numbers a, b, c are in G.P. if $b^2=ac$
The n^{th} term of a geometric series is $t_n = ar^{n-1}$
Sum of n terms of a Geometric sequence

$$S_n = \frac{a(r^n - 1)}{r - 1}, \quad r > 1$$

Where, a is the first term, r is the common ratio
Sum of terms of an infinite geometric progression,

$$S_\infty = \frac{a}{1-r}, r < 1.$$

Example 11: Which term of the series 2, 5, 8, 11, 14…. is 266?

(1) 66^{th} (2) 89^{th}
(3) 90^{th} (4) 92^{th}

Explanation:

Looking at the sequence, we can find that,

1^{st} term is 2 i.e. 3 x 1 – 1
2^{nd} term is 5 i.e. 3 x 2 – 1
3^{rd} term is 8 i.e. 3 x 3 – 1

So, 266 is $\frac{266+1}{3}$ i.e. 89^{th} term

Or

Use this formula, $t_n = a + (n-1)\,d$

Where a=2, t_n=266, d=3
266=2+(n-1)3
266-2=3n-3
264+3=3n
267=3n
n=89

Answer (2)

1.12 CSIR NET Solved Problems:

(CSIR NET/JRF DEC 2012)

1. Which of the following numbers is the largest?
$2^{3^4}, 2^{4^3}, 3^{2^4}, 3^{4^2}, 4^{2^3}, 4^{3^2}$

(1) 2^{3^4} (2) 3^{4^2}
(3) 4^{3^2} (4) 4^{2^3}

Explanation:

Consider the given options:
$2^{3^4} = 2^{81}$,
$3^{4^2} = 3^{16}$,
$4^{3^2} = 4^9$,
$4^{2^3} = 4^8$

We can clearly say that $4^8 < 4^9$. So, option 3 can be eliminated from the above-considered options.

To compare the remaining 3 numbers, 2^{81}, 3^{16}, 4^9 we need to make base or power of each number the same. So we can try to make the bases same.

$2^{81} = 2 \times 2^{80} = 2 \times (2^2)^{80} = 2 \times 4^{40}$

Thus, our remaining three numbers become 2×4^{40}, 3^{16}, 4^9.

Notice that the base and power of 4^{40} is greater than 3^{16} and 4^9, 4^{40} is greater than both the other numbers. Then surely, 2 times 4^{40} is also greater than both the other numbers.

Hence, 2^{81} is the largest.

Answer (1)

(CSIR NET/JRF DEC 2012)

2. Which of the following curve just touches the x-axis?

(1) $y = x^2 - x + 1$ (2) $y = x^2 - 2x + 2$
(3) $y = x^2 - 10x + 25$ (4) $y = x^2 - 7x + 12$

Explanation:

Clearly $x^2 - 10x + 25=0$
$(x - 5)^2 = 0$
x = 5, 5, real and equal roots.

Answer (3)

(DECEMBER 2012)

3. Consider the following equation:
$x^2 + 4y^2 + 9z^2 = 14x + 28y + 42z + 147$
Where, x, y and z are real numbers. Then the value of $x+2y+3z$ is

(1) 7 (2) 14
(3) 21 (4) not unique

Explanation:

$x^2+4y^2+9z^2 = 14x + 28y + 42z -147$

$x^2+4y^2+9z^2-14x-28y-42z+ 147=0$

$x^2+4y^2 +9z^2-14x-28y-42z+49 +49 +49=0$

$x^2-14x+49 +4y^2-28y + 49+ 9 z^2-42z+49=0$

$(x^2-14x+49) + (4y^2-28y+49) + (9z^2-42z+49)=0$

$(x-7)^2 +(2y-7)^2 \ (3 z -7)^2 = 0$

Thus, each term must be separately zero.

$(x-7)^2=0$

Take square root on both sides,

$(x-7)=0$

$x = 7$

$(2y-7)^2=0$

Take square root on both sides.

$(2y-7)=0$

$y = 7/2$

As, $(3 z -7)^2$

$Z = 7/3$

$x+2y+3z=7+2\left(\frac{7}{2}\right)+3\left(\frac{7}{3}\right) = 7+7+7=21$

Then, the value of x + 2y + 3z is 21

Answer (3)

(CSIR NET/JRF DEC 2012)

4. Consider the following equation:

$x^2 + 4y^2 + 9z^2 = 14x + 28y + 42z - 147$

Where x, y and z are real numbers.

Then the value of x + 2y + 3z is

(1) 7 (2) 14

(3) 21 (4) not unique

Explanation:

Given:

$x^2 + 4y^2 + 9z^2 = 14x + 28y + 42z – 147$

Grouping like terms together as below:

$(x^2 - 14x) + ((2y)^2- 28y) +((3z)^2 – 42z) + 147 = 0$

Making perfect squares by adding adequate numbers in each grouping as below:

$(x^2 - 14x + 49) + ((2y)^2- 28y + 49) +((3z)^2 – 42z + 49) = 0$

$(x – 7)^2 + (2y – 7)^2 + (3z – 7)^2 =0.$

The left-hand side in above equation is the sum of 3 non-negative numbers because every square number is positive.

For the sum of three non-negative numbers to be zero, they all should be zero.

So: $(x – 7)^2 = 0, (2y – 7)^2 = 0, (3z – 7)^2 = 0.$

$(x – 7) = 0, (2y – 7) = 0, (3z – 7) = 0.$

$x = 7, y = 7/2, z = 7/3.$

Thus, $x + 2y + 3z = 7 + (2×7/2) + (3×7/3) = 7 + 7 + 7= 21.$

Answer (3)

(CSIR NET/JRF JUNE 2013)

5. n is a natural number. If n^5 is odd, which of the following is true?

A. n is odd

B. n^3 is odd

C. n^4 is even

(1) A only (2) B only

(3) C only (4) A and B only

Explanation:

Consider any small natural number say 5, such that its fifth power is odd.

$5^5 = 3125$, odd

$5^4 = 625$, odd

$5^3 = 125$, odd

Clearly, only A and B are correct. Hence, option (4).

Answer (4)

(CSIR NET/JRF JUNE 13)

6. What is the last digit of 7^{73}?

(1) 7 (2) 9

(3) 3 (4) 1

Explanation:

To obtain the last digit of $7^2, 7^3$, …, multiply the last digits i.e., 7×7, 9×7, 3×7 etc.

So, the last digit is given by as below:

For 7^1, Last digit = 7, i.e., 7

For 7^2, Last digit = 7 × 7 = 49, i.e., 9

For 7^3, Last digit = 9 × 7 = 63, i.e., 3

For 7^4, Last digit = 3 × 7 = 21, i.e., 1.

For 7^5, Last digit = 1 × 7 = 7, i.e., 7

For 7^6, Last digit = 7 × 7 = 49, i.e., 9

For 7^7, Last digit = 9 × 7 = 63, i.e., 3

For 7^8, Last digit = 3 × 7 = 21, i.e., 1.

Note that {7, 9, 3, 1} is the cycle of last digits which gets repeated.

So, for powers, 1, 5, 9, 13… the last digit will be 7 itself.

For powers, 2, 6, 10, 14… the last digit will be 9.

For powers, 3, 7, 11, 15… the last digit will be 3.

For powers, 4, 8, 12, 16… the last digit will be 1.

In the question, the power is given as 73.

73 when divided by 4, gives a remainder 1.

So, 73 should be in the series 1, 5, 9, 13…

Thus, the last digit is 7.

Answer (1)

(CSIR NET/JRF JUNE 2013)

7. Suppose you expand the product $(x_1 + y_1)(x_2 + y_2)…(x_{20} + y_{20})$. How many terms will have only one x and rest y's?

(1) 1 (2) 5

(3) 10 (4) 20

Explanation:

Here,

$(x_1 + y_1)$ ----- 1 term

$(x_1 + y_1)(x_2 + y_2) = (x_1 x_2 + x_1 y_2 + x_2 y_1 + y_1 y_2)$ ----- 2 terms

Continuing like this, we can find that if 20 factors are there in the product, there will be 20 terms in the expression with only 1 x and rest all y's.

Answer (4)

(CSIR NET/JRF JUNE 2013)

8. In solving a quadratic equation of the form $x^2 + ax + b = 0$, one student took the wrong value of a and got the roots as 6 and 2; while another student took the wrong value of b and got the roots as 6 and 1.

What are the correct values of a and b, respectively?

(1) 7 and 12 (2) 3 and 4
(3) -7 and 12 (4) 8 and 12

Explanation:

For a general quadratic equation: $mx^2 + nx + p = 0$,

Sum of roots = -n/m, Product of roots = p/m

Given: $x^2 + ax + b = 0$

So, m = 1, n = a, p = b.

Therefore, sum of roots = -a

Product of roots = b

Given: First student took the wrong value of a, but the correct value of b.

So, the product of roots is correct.

Therefore, product of roots (b) = 6 × 2 = 12.

Given: Another student took the wrong value of b, but correct value of a.

So, the sum of roots is correct.

Therefore, sum of roots (-a) = 6 + 1 = 7

-a = 7

a = -7

So, the correct values of a and b are -7 and 12.

Answer (3)

(CSIR NET/JRF JUNE 2013)

9. Choose the largest number:

(1) 2^{500} (2) 3^{400}
(3) 4^{300} (4) 5^{200}

Explanation:

If we have to compare these numbers, then we should either make the base same, or the power same.

Here in the base, the numbers are 2, 3, 4 and 5. It is difficult to make the base same here.

Therefore, we can try making the exponent same.

$2^{500} = (2^5)^{100} = 32^{100}$,

$3^{400} = (3^4)^{100} = 81^{100}$,

$4^{300} = (4^3)^{100} = 64^{100}$,

$5^{200} = (5^2)^{100} = 25^{100}$.

Now the exponents are all the same.

Therefore, the number with largest base is the largest number.

Hence, $81^{100} = 3^{400}$ is the largest number.

Answer (2)

(CSIR NET/JRF JUNE 2013)

10. Define

$a \otimes b$ = LCM (a, b) + GCD (a, b) and $a \oplus b = a^b + b^a$.

What is the value of $(1 \oplus 2) \otimes (3 \oplus 4)$?

Here LCM = least common multiple and GCD = greatest common divisor.

(1) 145 (2) 286
(3) 436 (4) 572

Explanation:

Given: $a \oplus b = a^b + b^a$

So, $1 \oplus 2 = 1^2 + 2^1 = 1 + 2 = 3$,

$3 \oplus 4 = 3^4 + 4^3 = 81 + 64 = 145$

Therefore, $(1 \oplus 2) \otimes (3 \oplus 4) = 3 \otimes 145$

It is said that $a \otimes b$ = LCM (a, b) + GCD (a, b)

So, $(1 \oplus 2) \otimes (3 \oplus 4)$ = LCM (3,145) + GCD (3,145)

= 435 + 1 = 436.

Answer (3)

(CSIR NET/JRF JUNE 2013)

11. How many pairs of positive integers have GCD 20 and LCM 600? (GCD = greatest common divisor, LCM = least common multiple)

(1) 4 (2) 0
(3) 1 (4) 7

Explanation:

Note:

For any two number (p and q):

GCD (p, q) × LCM (p, q) = p × q

Given: GCD = 20 & LCM = 600

20 × 600 = p × q

i.e. p × q =12000.

Since p and q have GCD 20, both p and q are multiples of 20.

i.e., p = 20x, q= 20y,

where x and y are relatively prime numbers.

So, 20x × 20y = 12000

400x × y = 12000

x × y = 30.

So, possible combinations of x and y are

x	1	2	3	5
y	30	15	10	6

Thus, possible pairs of such positive integers with GCD 20 and LCM 600 are as below:

20 × 1, 20 × 30 = 20, 600

20 × 2, 20 × 15 = 40, 300

20 × 3, 20 × 10 = 60, 200

20 × 5, 20 × 6 = 100, 120

Thus, there are 4 pairs.

Answer (1)