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Section 1

Engineering Mathematics

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1 CHAPTER

LINEAR ALGEBRA

MULTIPLE CHOICE QUESTIONS (MCQ)

1. If 3A – B = $\begin{bmatrix}5 & 0\\1 & 1\end{bmatrix}$ and B = $\begin{bmatrix}4 & 3\\2 & 5\end{bmatrix}$ then find the value of matrix A.
   (1) $\begin{bmatrix}3 & 2\\1 & 1\end{bmatrix}$ (2) $\begin{bmatrix}3 & 1\\2 & 1\end{bmatrix}$
   (3) $\begin{bmatrix}3 & 1\\1 & 2\end{bmatrix}$ (4) $\begin{bmatrix}1 & 2\\3 & 1\end{bmatrix}$

2. The eigen values of A= $\begin{bmatrix}8 & 2\\0 & -4\end{bmatrix}$ are
   (1) 2, -4 (2) 4, -8
   (3) 8, -4 (4) 4, -4

3. The determinant of the matrix $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\6 & 7 & 8\end{bmatrix}$ is
   (1) 1 (2) 0
   (3) -1 (4) 3

4. Write the number of all possible matrices of order 2 × 2 with each entry 1, 2 or 3.
   (1) 32 (2) 49
   (3) 36 (4) 81

5. Matrix A = $\begin{bmatrix}0 & -2\\p & 0\end{bmatrix}$ will be skew-symmetric when p=?
   (1) -2 (2) 1/2
   (3) -1/2 (4) 2

6. If A = $\begin{bmatrix}3 & 2\\5 & 4\end{bmatrix}$, then $A^2 + 2A$ will be
   (1) $\begin{bmatrix}25 & 18\\35 & 34\end{bmatrix}$
   (2) $\begin{bmatrix}15 & 18\\45 & 34\end{bmatrix}$
   (3) $\begin{bmatrix}25 & 18\\45 & 34\end{bmatrix}$
   (4) $\begin{bmatrix}25 & 45\\18 & 34\end{bmatrix}$

7. What is rank of the matrix?
   $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}$
   (1) 1 (2) 2
   (3) 3 (4) 4

8. The system of linear equation
   $ax + y = 2$
   $2x + 3y = 5$
   The system of linear equations in two variables shown above will have unique solutions, what is value of a?
   (1) 1 (2) 2
   (3) 3 (4) 4

9. What are the eigenvalues of the following matrix?
   $\begin{bmatrix}3 & 2\\5 & 4\end{bmatrix}$
   (1) $\frac{3\pm\sqrt{41}}{2}$ (2) $\frac{5\pm\sqrt{31}}{2}$
   (3) $\frac{5\pm\sqrt{41}}{2}$ (4) $\frac{3\pm\sqrt{31}}{2}$

10. Which of the following matrix has eigen value 2, 3
    (1) $\begin{bmatrix}2 & 1\\0 & -3\end{bmatrix}$ (2) $\begin{bmatrix}2 & 0\\5 & -2\end{bmatrix}$
    (3) $\begin{bmatrix}2 & 1\\0 & 3\end{bmatrix}$ (4) $\begin{bmatrix}2 & 0\\3 & -2\end{bmatrix}$

11. If A is square matrix then orthogonality property mandates
    (1) $AA^T = I$ (2) $AA^T = 0$
    (3) $AA^T = A^{-1}$ (4) $AA^T = A^2$

12. If P= $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ and Q = $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ then $Q^T P^T$ is
    (1) $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ (2) $\begin{bmatrix}1 & 3\\2 & 4\end{bmatrix}$
    (3) $\begin{bmatrix}2 & 1\\4 & 3\end{bmatrix}$ (4) $\begin{bmatrix}2 & 4\\1 & 3\end{bmatrix}$

13. The matrix P given below
    P=$\begin{bmatrix}0 & 1 & 3 & 0\\-2 & 3 & 0 & 4\\0 & 0 & 6 & 1\\0 & 0 & 1 & 6\end{bmatrix}$ the eigen value of P is given by
    (1) 0,3,6,6
    (2) 1,2,3,4
    (3) 3,4,5,7
    (4) 1,2,5,7

NUMERICAL ANSWER TYPE (NAT)

1. The negative eigen value of the following matrix is _____ (in integer)
$\begin{bmatrix} 2 & 1 \\ 5 & -2 \end{bmatrix}$

2. The determinant of the matrix $\begin{bmatrix} 2021 & 2022 \\ 2023 & 2024 \end{bmatrix}$ is _____ (in integer)

3. The largest eigen value of the matrix $\begin{bmatrix} 3 & 2 \\ -1 & 6 \end{bmatrix}$ is _____(in integer)

4. If A = $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$, then determinant of the A^3 is _____ (in integer)

5. If the sum & product of eigen values of 2×2 real matrix $\begin{bmatrix} 3 & p \\ p & q \end{bmatrix}$ are 4& -1 respectively, then $|p|$ is _____ (in integer)

6. If $\begin{bmatrix} 10 & 2k+5 \\ 3k-3 & k+5 \end{bmatrix}$ is symmetric matrix, the value of k is _____(in integer)

7. The rank of the matrix, M = $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ is _____

ANSWER KEY									
MULTIPLE CHOICE QUESTIONS (MCQ)									
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
(3)	(3)	(2)	(4)	(4)	(3)	(2)	(1)	(3)	(3)
11	**12**	**13**							
(1)	(4)	(4)							
NUMERICAL ANSWER TYPE (NAT)									
1	**2**	**3**	**4**	**5**	**6**	**7**			
-3	-2	5	1000	2	8	3			

SOLUTION

MULTIPLE CHOICE QUESTIONS (MCQ)

1. Correct Answer is (3):

Given

$3A - B = \begin{bmatrix}5 & 0\\ 1 & 1\end{bmatrix}, B = \begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix}$

$\Rightarrow 3A - \begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix} = \begin{bmatrix}5 & 0\\ 1 & 1\end{bmatrix}$

$\Rightarrow 3A = \begin{bmatrix}5 & 0\\ 1 & 1\end{bmatrix} + \begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix}$

$\Rightarrow 3A = \begin{bmatrix}5+4 & 0+3\\ 1+2 & 1+5\end{bmatrix}$

$\Rightarrow 3A = \begin{bmatrix}9 & 3\\ 3 & 6\end{bmatrix}$

$\Rightarrow A = \frac{1}{3}\begin{bmatrix}9 & 3\\ 3 & 6\end{bmatrix}$

$\Rightarrow A = \begin{bmatrix}3 & 1\\ 1 & 2\end{bmatrix}$

⇒ option (3) is correct.

2. Correct Answer is (3):

Given, $A = \begin{bmatrix}8 & 2\\ 0 & -4\end{bmatrix}$

By observing, matrix A is upper triangular matrix

So, diagonal entries are the required eigen value.

⇒ 8, -4 are the eigen value of matrix A.

⇒ option (3) is correct.

3. Correct Answer is (2):

Given matrix,

$A = \begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 6 & 7 & 8\end{bmatrix}$

$|A|$ = 1(40 – 42) -2(32 – 36) + 3(28 – 30)

= 1(-2) -2(-4) +3(-2)

= -2 + 8 -6

= 0

⇒ option (2) is correct.

4. Correct Answer is (4): We know that, a matrix of order 2 × 2 has 4 entries. Since, each entry has 3 choices, namely 1, 2 or 3, therefore number of required matrices

3^4 = 3 × 3 × 3 × 3 = 81.

⇒ option (4) is correct.

5. Correct Answer is (4):

Matrix A is skew-symmetric

⇒ A = -A^T

$\Rightarrow \begin{bmatrix}0 & -2\\ p & 0\end{bmatrix} = -\begin{bmatrix}0 & p\\ -2 & 0\end{bmatrix}$

$\Rightarrow \begin{bmatrix}0 & -2\\ p & 0\end{bmatrix} = \begin{bmatrix}0 & -p\\ 2 & 0\end{bmatrix}$

⇒ p= 2

⇒ option (4) is correct.

6. Correct Answer is (3):

Given, A = $\begin{bmatrix}3 & 2\\ 5 & 4\end{bmatrix}$

$A^2 + 2A = \begin{bmatrix}3 & 2\\ 5 & 4\end{bmatrix}\begin{bmatrix}3 & 2\\ 5 & 4\end{bmatrix} + 2\begin{bmatrix}3 & 2\\ 5 & 4\end{bmatrix}$

$= \begin{bmatrix}9+10 & 6+8\\ 15+20 & 10+16\end{bmatrix} + \begin{bmatrix}6 & 4\\ 10 & 8\end{bmatrix}$

$= \begin{bmatrix}19 & 14\\ 35 & 26\end{bmatrix} + \begin{bmatrix}6 & 4\\ 10 & 8\end{bmatrix}$

$= \begin{bmatrix}25 & 18\\ 45 & 34\end{bmatrix}$

⇒ option (3) is correct.

7. Correct Answer is (2):

Given,

$\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{bmatrix}$

We take $R_3 \to R_3 - 7R_1$ &$R_2 \to R_2 - 4R_1$

$\begin{bmatrix}1 & 2 & 3\\ 0 & -3 & -6\\ 0 & -6 & -12\end{bmatrix}$

Next $R_3 \to R_3 - 2R_2$

$\begin{bmatrix}1 & 2 & 3\\ 0 & -3 & -6\\ 0 & 0 & 0\end{bmatrix}$

$R_2 \to -R_2/3$

$\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 2\\ 0 & 0 & 0\end{bmatrix}$

Rank = number of non-zero row in row echelon form = 2.

⇒ option (2) is correct.

8. Correct Answer is (1):

Given,

$ax + y = 2 \dots\dots\dots (1)$

$2x + 3y = 5 \dots\dots\dots (2)$

Multiplying equation (1) by 3 we get

$3ax + 3y = 6 \dots\dots\dots (3)$

Solve (3) in (2) we get,

$x = 1, y = 1$

Put in equation (1)

$\Rightarrow a + 1 = 2 \quad \Rightarrow a$ = 1

⇒ option (1) is correct.

9. Correct Answer is (3):

Given, A= $\begin{bmatrix}3 & 2\\ 5 & 4\end{bmatrix}$

Consider $|A - \lambda I|$ = 0

$\left|\begin{bmatrix}3 & 2\\ 5 & 4\end{bmatrix} - \lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right|$ = 0

$\begin{vmatrix}3-\lambda & 2\\ 5 & 2-\lambda\end{vmatrix}$ = 0

$(3 - \lambda)(2 - \lambda) - 10 = 0$

$\lambda^2 - 3\lambda - 2\lambda + 6 - 10 = 0$

$\lambda^2 - 5\lambda - 4 = 0$

$\lambda = \frac{5 \pm \sqrt{41}}{2}$

⇒ option (3) is correct.

10. Correct Answer is (3): We know that every upper triangular or lower triangular matrix eigen value are diagonal entries

So, matrix $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ has eigen value 2,3.

Therefore, option (3) is correct.

11. Correct Answer is (1):

If, $AA^T = I \; or \;\; A^T = A^{-1}$

The matrix is orthogonal.

⇒ option (1) is correct.

12. Correct Answer is (4):

Given, P= $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and Q = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

PQ = $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

$Q^T P^T = (PQ)^T = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

⇒ option (4) is correct.

13. Correct Answer is (4):

$|P|$ = 70 and trace(P) = 15

So, only (1,2,5,7) satisfies.

⇒ option (4) is correct.

NUMERICAL ANSWER TYPE (NAT)

1. Correct Answer is (-3):

Given, matrix A = $\begin{bmatrix} 2 & 1 \\ 5 & -2 \end{bmatrix}$

$|A - \lambda I| = 0$

$\left|\begin{bmatrix} 2 & 1 \\ 5 & -2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right|$ = 0

$\begin{vmatrix} 2-\lambda & 1 \\ 5 & -2-\lambda \end{vmatrix}$ = 0

$(2 - \lambda)(-2 - \lambda) - 5 = 0$

$\lambda^2 - 9 = 0, \lambda = \pm 3$

Therefore, the negative eigen value is -3.

2. Correct Answer is (-2):

Given, matrix A = $\begin{bmatrix} 2021 & 2022 \\ 2023 & 2024 \end{bmatrix}$

$\Rightarrow \; |A| = 2021(2024) - (2023)(2022)$

= 4090504 – 4090506 = -2

3. Correct Answer is (5):

Given, matrix A = $\begin{bmatrix} 3 & 2 \\ -1 & 6 \end{bmatrix}$

$|A - \lambda I| = 0$

$\left|\begin{bmatrix} 3 & 2 \\ -1 & 6 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right|$ = 0

$\begin{vmatrix} 3-\lambda & 2 \\ -1 & 6-\lambda \end{vmatrix}$ = 0

$(3 - \lambda)(6 - \lambda) + 2 = 0$

$\lambda^2 - 9\lambda + 18 + 2 = 0$

$\lambda^2 - 9\lambda + 20 = 0 \Rightarrow \lambda = 4,5$

⇒ the largest eigen value of the matrix is 5

4. Correct Answer is (1000):

Given, A = $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$

$A^3 = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 18 & 14 \\ 7 & 11 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 72+14 & 36+42 \\ 28+11 & 14+33 \end{bmatrix}$

$A^3 = \begin{bmatrix} 86 & 78 \\ 39 & 47 \end{bmatrix} \Rightarrow |A^3| = 86(47) - 39(78)$

= 4042 – 3042 = 1000.

⇒ therefore, the determinant of A^3 is 1000.

5. Correct Answer is (2):

From the property of eigen values,

Sum of eigenvalues = trace of matrix

4 = 3+q

q = 1 product of eigen value = determinant

-1 = $\begin{vmatrix} 3 & p \\ p & q \end{vmatrix}$

$3q - p^2 = -1$

$3 - p^2 = -1$

$p^2 = 4$, p = ±2 ⇒|p| = 2

6. Correct Answer is (8):

A = $\begin{bmatrix} 10 & 2k+5 \\ 3k-3 & k+5 \end{bmatrix}$

⇒ (2k+ 5) = (3k – 3)

⇒ k = 8.

7. Correct Answer is (3):

M = $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$R_1 \leftrightarrow R_2$

M = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$R_3 \rightarrow R_3 - R_1$

M = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$

$R_3 \rightarrow R_3 - R_2$

= $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix}$

Which is in echelon form

$\rho(M) = 3$.