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Section 1

Engineering Mathematics

Chapter 1: Linear Algebra: Matrices and determinants; Systems of linear equations; Eigen values and Eigen vectors.

Chapter 2: Calculus: Limits, continuity and differentiability; Partial derivatives, maxima and minima; Sequences and series; Test for convergence.

Chapter 3: Differential Equations: Linear and nonlinear first order ODEs, higher order ODEs with constant coefficients; Cauchy's and Euler's equations; Laplace transforms.

Chapter 4: Probability and Statistics: Mean, median, mode and standard deviation; Random variables; Poisson, normal and binomial distributions; Correlation and regression analysis.

Chapter 5: Numerical Methods: Solution of linear and nonlinear algebraic equations; Integration by trapezoidal and Simpson's rule; Single step method for differential equations.

ENGINEERING MATHEMATICS: NUMBER OF QUESTION ASKED IN GATE EXAM

	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Linear Algebra	1	1	2	1	4	2	2	1	1	2	2
Calculus	-	-	1	2	2	1	1	1	2	3	2
Differential Equation	-	-	3	3	-	2	1	-	4	2	2
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1 CHAPTER

LINEAR ALGEBRA

(2012)

1. What is the rank of the following matrix?

$$\begin{pmatrix} 5 & 3 & -1 \\ 6 & 2 & -4 \\ 14 & 10 & 0 \end{pmatrix}$$

(A) 0 (B) 1

(C) 2 (D) 3

(2019)

2. Matrix A = $\begin{bmatrix} 0 & 6 \\ p & 0 \end{bmatrix}$ will be skew-symmetric when p = ____________

(2018)

3. The determinant of the matrix $\begin{pmatrix} 4 & -6 \\ -3 & 2 \end{pmatrix}$ is ____________

(2013)

4. If $P = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, $Q = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$, *and* $R = \begin{bmatrix} 3 & 0 \\ 1 & 3 \end{bmatrix}$, which one of the following statements is TRUE ________________

(A) PQ = PR (B) QR= RP

(C) QP = RP (D) PQ = QR

(2017)

5. The value of c for which the following system of linear equations has an infinite number of solutions is______________

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ 4 \end{bmatrix}$$

(2016)

6. The value of determinant A given below is ________________

$$A = \begin{pmatrix} 5 & 16 & 81 \\ 0 & 2 & 2 \\ 0 & 0 & 16 \end{pmatrix}$$

(2011)

7. Value of the determinant mentioned below is ________________

$$\begin{vmatrix} 1 & 0 & -1 & 0 \\ 4 & 7 & 0 & 2 \\ 1 & 1 & -1 & 1 \\ 2 & 0 & 2 & 1 \end{vmatrix}$$

(A) +24 (B) -30

(C) -24 (D) -10

(2015)

8. The determinant of the matrix $\begin{bmatrix} 3 & 0 & 0 \\ 2 & 5 & 0 \\ 6 & -8 & -4 \end{bmatrix}$ is ____________

(2021)

9. The determinant of matrix $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 3 \end{pmatrix}$ is __________

(2021)

10. If the area of a triangle with the vertices $(k, 0)$, (2,0) and (0,-2) is 2 square units, the value of k is__________.

(2015)

11. If A = $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$, then $A^2 + 3A$ will be ________________

(A) $\begin{bmatrix} 30 & 20 \\ 10 & 20 \end{bmatrix}$ (B) $\begin{bmatrix} 28 & 10 \\ 4 & 18 \end{bmatrix}$

(C) $\begin{bmatrix} 31 & 13 \\ 7 & 21 \end{bmatrix}$ (D) $\begin{bmatrix} 20 & 10 \\ 5 & 15 \end{bmatrix}$

(2015)

12. $2x_1 + x_2$ = 3

$5x_1 + bx_2$ = 7.5

The system of linear equations in two variables shown above will have infinite solutions, if and only if, b is equal to ______________

(2020)

13. The system of linear equation

cx + y = 5

3x + 3y – 6

has no solution when e is equal to _______________

(2015)

14. What are the eigenvalues of the following matrix?

$$\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$$

(A) 2 and 3 (B) -2 and 3

(C) 2 and 3 (D) -2 and -3

(2020)

15. The largest eigenvalue of the matrix $\begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix}$ is ______________

(2014)

16. The eigenvalues of A = $\begin{bmatrix} 1 & -4 \\ 2 & -3 \end{bmatrix}$ are ________________

(A) 2 $\pm I$ (B) -1, -2

(C) -1$\pm 2i$ (D) non-existent

(2013)

17. One of the eigen values of

$P = \begin{bmatrix} 10 & -4 \\ 18 & -12 \end{bmatrix}$ is

(A) 2 (B) 4

(C) 6 (D) 8

(2016)

18. The positive Eigen value of the following matrix is____________

$\begin{bmatrix} 2 & 1 \\ 5 & -2 \end{bmatrix}$

(2017)

19. The angle (in degrees) between the vectors $\vec{x} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{y} = 2\hat{\imath} - \hat{\jmath} - 1.5\hat{k}$ is _____________

ANSWERS

1. (c)	**2.** (6)	**3.** (10)	**4.** (d)	**5.** (4)	**6.** (160)	**7.** (10)	**8.** (60)	**9.** (8)	**10.** (0 or 4)
11. (A)	**12.** (2.5)	**13.** (1)	**14.** (A)	**15.** (3)	**16.** (c)	**17.** (c)	**18.** (3)	**19.** (90°)	

EXPLANATION

1. Given matrix

$$A = \begin{pmatrix} 5 & 3 & -1 \\ 6 & 2 & -4 \\ 14 & 10 & 0 \end{pmatrix}$$

$R_2 \to 5R_2 - 6R_1, R_3 \to 5R_3 - 14R_1$

$$= \begin{pmatrix} 5 & 3 & -1 \\ 0 & -8 & -14 \\ 0 & 8 & 14 \end{pmatrix}$$

$R_3 \to R_3 + R_2$

$$= \begin{pmatrix} 5 & 3 & -1 \\ 0 & -8 & -14 \\ 0 & 0 & 0 \end{pmatrix}$$

Rank = Number of non-zero row in row echelon form $= 2$

2. Matrix A is skew-symmetric

$\Rightarrow A = -A^T$

$\Rightarrow \begin{bmatrix} 0 & 6 \\ p & 0 \end{bmatrix} - \begin{bmatrix} 0 & p \\ 6 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & 6 \\ p & 0 \end{bmatrix} = \begin{bmatrix} 0 & -p \\ -6 & 0 \end{bmatrix}$

$\Rightarrow p = 6$

3. Given matrix $A = \begin{pmatrix} 4 & -6 \\ -3 & 2 \end{pmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 4 & -6 \\ -3 & 2 \end{vmatrix}$

$= 4 \times 2 - (-6) \times (-3)$

$= 8 - 18 = -10$

4. $p = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}, Q = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}, R = \begin{bmatrix} 3 & 0 \\ 1 & 3 \end{bmatrix}$

$PQ = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2+2 & 1+2 \\ 4+4 & 2+4 \end{bmatrix}$

$= \begin{bmatrix} 4 & 3 \\ 8 & 6 \end{bmatrix}$

$PR = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3+1 & 3 \\ 6+2 & 6 \end{bmatrix}$

$= \begin{bmatrix} 4 & 3 \\ 8 & 6 \end{bmatrix}$

$\Rightarrow PQ = PR$

5. Given system of equation

$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ 4 \end{bmatrix}$

$\Rightarrow x + 2y = c$

$x + 2y = 4$

Note:- Consider a system

$a_1x + b_1y = c_1$

$a_2x + b_2y = c_2$

Then above system has infinite number of explanation if

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

From (*)

$\frac{1}{1} = \frac{2}{2} = \frac{c}{4} \Rightarrow 1 = \frac{c}{4} \Rightarrow c = 4$

6. Given matrix $A = \begin{pmatrix} 5 & 16 & 81 \\ 0 & 2 & 2 \\ 0 & 0 & 16 \end{pmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 5 & 16 & 81 \\ 0 & 2 & 2 \\ 0 & 0 & 16 \end{vmatrix}$

$= 5(2 \times 16 - 0 \times 2) - 16(0 \times 16 - 0 \times 2) + 81(0 - 2 \times 0)$

$= 5(32) - 16(0) + 81(0)$

$= 160$

7. Given matrix $A = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 4 & 7 & 0 & 2 \\ 1 & 1 & -1 & 1 \\ 2 & 0 & 2 & 1 \end{bmatrix}$

$|A| = 1\begin{vmatrix} 7 & 0 & 2 \\ 1 & -1 & 1 \\ 0 & 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 4 & 7 & 2 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix}$

$= 7(1-2) - 0 + 2(2-0) - [4(1-0) - 7(1-2) + 2(0-2)]$

$= 7(=1) + 4 - (4 + 7 - 4)$

$= -4 + 4 - 4 - 7 + 4$

$= -10$

8. Given matrix $A = \begin{pmatrix} 3 & 0 & 0 \\ 2 & 5 & 0 \\ 6 & -8 & -4 \end{pmatrix}$

$|A| = 3(5 \times 1 - 4) - (-8) \times 0) - 0 + 0$

$3 \times 5 \times (-4)$

$= -60$

9.

$\begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 3 \end{vmatrix}$

Applying $\mathrm{R_2 + R_1;\ R_3 + R_1;\ R_4 - R_1}$)

$= \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 2 \end{vmatrix}$

is determinant of upper triangular matrix $= 1 \times 2 \times 2 \times 2 = 8$

10. Area of triangle with the vertices $\mathrm{A(x_1, y_1), B\ (x_2, y_2) and\ C\ (x_3, y_3) is}$

$\left|\frac{1}{2}\begin{vmatrix} 1 & \mathrm{x_1} & \mathrm{y_1} \\ 1 & \mathrm{x_2} & \mathrm{y_2} \\ 1 & \mathrm{x_3} & \mathrm{y_3} \end{vmatrix} = 2\right|$

$\Rightarrow \left| \begin{array}{c} \frac{1}{2}\begin{vmatrix} 1 & \mathrm{k} & 0 \\ 1 & 2 & 0 \\ 1 & 0 & -2 \end{vmatrix} = 2 \Rightarrow \\ \left|\frac{1}{2}\{-2(2-\mathrm{k})\}\right| = 2 \Rightarrow |\mathrm{k} - 2| \\ = 2 \Rightarrow \mathrm{k} - 2 = \pm 2 \end{array} \right|$

$\therefore \mathrm{k} = 0 \text{ or } 4 \text{ (both correct)}$

11. 52. $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ then

$A^2 + 3A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} + 3\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 16+2 & 8+6 \\ 4+3 & 2+9 \end{bmatrix} + B\begin{bmatrix} 12 & 6 \\ 3 & 9 \end{bmatrix}$

$= \begin{bmatrix} 18 & 14 \\ 7 & 11 \end{bmatrix} + \begin{bmatrix} 12 & 6 \\ 3 & 9 \end{bmatrix}$

$= \begin{bmatrix} -30 & 20 \\ 10 & 20 \end{bmatrix}$

12. $2x_1 + x_2 = 3$

$5x_1 + bx_2 = 7.5$

Above system has infinite no. of solution if

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{5} = \frac{1}{b} = \frac{3}{7.5}$

$\Rightarrow \frac{2}{5} = \frac{1}{b} \Rightarrow 2b = 5 \Rightarrow b = \frac{5}{2}$

$\therefore b = 2.5$

13. Given system of equation

$Cx + y = 5$

$3x + 3y = 6$

Note:- Consider the system

$a_1x + b_1y = c_1$

$a_2x + b_2y = c_2$

Then above system has no solution if

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

System (*) has no solution

$\Rightarrow \frac{c}{3} = \frac{1}{3} \neq \frac{5}{6} \Rightarrow c = 1$

14. Given matrix $A \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$

Consider $|A - \lambda I| = 0$

$\left|\begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix} - \lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right| = 0$

$\begin{vmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{vmatrix} = 0$

$(1-\lambda)(4-\lambda) + 2 = 0$

$\lambda^2 - 5\lambda + 6 = 0$

$\lambda^2 - 2\lambda - 3\lambda + 6 = 0$

$\lambda(\lambda - 2) - 3(\lambda - 2) = 0$

$(\lambda - 2)(\lambda - 3) = 0$

$\lambda = 2, \lambda = 3$

$\therefore$ Eigen values are 2 and 3

15. Given matrix $A\begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix}$

Consider $|A-\lambda I|=0$

$\left|\begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix}-\lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right|=0$

$\begin{vmatrix} 4-\lambda & 1 \\ -2 & 1-\lambda \end{vmatrix}=0$

$(4-\lambda)(1-\lambda)+2=0$

$\lambda^2-5\lambda+6=0$

$\lambda^2-3\lambda-2\lambda+6=0$

$\lambda(\lambda-3)-2(\lambda-3)=0$

$(\lambda-2)(\lambda-3)$

$\lambda=2,\quad 3$

∴ 2 and 3 are eigen value of A

⇒ Largest eigen value of the matrix A is 3

16. Given matrix $A=\begin{bmatrix} 1 & -4 \\ 2 & -3 \end{bmatrix}$

Consider $|A=\lambda I|=0$

$\left|\begin{pmatrix} 1 & -4 \\ 2 & -3 \end{pmatrix}-\lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right|=0$

$\Rightarrow \begin{vmatrix} 1-\lambda & -4 \\ 2 & -3-\lambda \end{vmatrix}=0$

$(1-\lambda)(-3-\lambda)+8=0$

$\lambda 2+2\lambda+5=0$

Now, we find roots by quadratic formula

$\lambda=\frac{-b\pm\sqrt{b^2-4ac}}{2q}=\frac{-2\pm\sqrt{z^2-4\times1\times5}}{2\times1}$

$=\frac{-2\pm\sqrt{-16}}{2}=\frac{-2\pm4\,i}{2}=-1\pm2\,i$

17. Given matrix $P=\begin{bmatrix} 10 & -4 \\ 18 & -12 \end{bmatrix}$

Consider $|P-\lambda I|=0$

$\left|\begin{pmatrix} 10 & -4 \\ 18 & 12 \end{pmatrix}-\lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right|=0$

$\begin{vmatrix} 10-\lambda & -4 \\ 18 & -12-\lambda \end{vmatrix}=0$

$(10-\lambda)(-12-\lambda)+72=0$

$\lambda^2+2\lambda-48=0$

$\lambda^2+8\lambda-6\lambda-48=0$

$\lambda(\lambda+8)-6(\lambda+8)=0$

$(\lambda+8)(\lambda-61)=0$

$\Rightarrow \lambda=-8,\lambda=6$

One of the eigen value = 6

18. Given matrix $A=\begin{bmatrix} 2 & 1 \\ 5 & -2 \end{bmatrix}$

Consider $|A-\lambda I|=0$

$\left|\begin{pmatrix} 2 & 1 \\ 5 & -2 \end{pmatrix}-\lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right|=0$

$\begin{vmatrix} 2-\lambda & 1 \\ 5 & -2-\lambda \end{vmatrix}=0$

$(2-\lambda)(-2-\lambda)-5=0$

$\lambda^2-9=0\quad \lambda=\pm3$

19. $\vec{x}=\hat{\imath}-\hat{\jmath}+2\hat{k}$

$\vec{y}=2\hat{\imath}-\hat{\jmath}+1.5\hat{k}$

$\cos\theta=\frac{\vec{x}.\vec{y}}{|\vec{x}|.|\vec{y}|}$

$=\frac{2+1-2(1.5)}{|\vec{x}|.|\vec{y}|}=0$

$\cos\theta=0$

$\Rightarrow\ \theta=90^0$

2 CHAPTER

CALCULUS

(2015)

1. The limit of the function $(1+\frac{x}{n})^{n}$ as $n \rightarrow \infty$ is ________________

(A) lnx (B) $\ln\frac{1}{x}$

(C) e^{-x} (D) e^{x}

(2014)

2. The limit of the function $e^{-2t}\sin(t) as\ t \rightarrow \infty, is$ _______________

(2019)

3. The solution of $\lim_{x \rightarrow 8}\left(\frac{x^2-64}{x-8}\right)$ is _______________

(2013)

4. Evaluate lim x tan1/x ________________

(A) ∞ (B) 1

(B) 0 (D) -1

(2017)

5. $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$ is ________________

(2021)

6. The value of $\lim_{x\rightarrow 0}\left[\frac{x-\sin 2x}{x-\sin 5x}\right]$(rounded off two decimal places) is _________________

(2020)

7. A function f is as follows

$$f(x) = \begin{cases} 15 & if\ x < 1 \\ cx & if\ x \geq 1 \end{cases}$$

The function f is a continuous function when c is equal to ________ (answer is an integer)

(2020)

8. A function f is given as

$$f = (X) = 4X - X^2$$

The function f is maximized when X is equal to _______________

(2016)

9. Consider the equation

$$V = \frac{aS}{b + S + \frac{S^2}{c}}$$

Given a = 4, b = 1 and c = 9, the positive value of S at which V is maximum, will be ______________

(2015)

10. Consider the following infinite series:

$1+r+ r^2+r^3+ \ldots\ldots.$

If r = 0.3, then the sum of this infinite series is______________

(2019)

11. Which of the following are geometric series? ________________

P. 1, 6, 11, 16, 21, 26, --- Q. 9, 6, 3, 0, -3, -6, ----

R. 1, 3, 9, 27, 81, ---- S. 4, -8, 16, -32, 64, -----

(A) P and Q only (B) R and S only

(C) Q and S only (D) P, Q and R only

(2014)

12. Which of the following statements is true for the series given below?

$$S_n = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots + \frac{1}{\sqrt{n}}$$

(A) S_n converges to $\log(\sqrt{n})$

(B) S_n converges to $\sqrt{n}$

(C) S_n converges to $\exp(\sqrt{n})$

(D) $S_n\ diverges$

(2020)

13. An infinite series S is given as:

S= 1 + 2/3 + 3/9 + 4/27 + 5/81 + ……(to infinity)

The value of S is_____________ (round off to 2 decimal places)

(2018)

14. If $1 + r + r^2 + r^3 + \cdots\infty = 1.5$, then, $1 + 2r + 3r^2 + 4r^3 + \cdots \infty =$ ____________ (up to two decimal places)

(2021)

15. The sum of the infinite geometric series $1 + {}^{1}/_{3} + {}^{1}/_{3^2} + {}^{1}/_{3^3} + \ldots$ (rounded off to one decimal place) is__________.

ANSWERS

1. (D)	**2.** (0)	**3.** (16)	**4.** (B)	**5.** (1)
6. (0.25)	**7.** (15)	**8.** (2)	**9.** (± 3)	**10.** (1.4285)
11. (B)	**12.** (D)	**13.** (-3.87 to -3.38)	**14.** (2.25)	**15.** (1.5 to 1.5)

EXPLANATION

1. If $f(n) = [9(n)]^{h(n)}$ then if $\lim_{x\to\infty} f(n)$ is of the form $(1)^\infty$.

Then $\lim_{x\to\infty} f(n) = e^{\lim_{x\to\infty} h(n)[9(n)-1]}$

Here $9(n) = \left(1+\frac{x}{n}\right)$ & $h(n) = n$

$\therefore \lim_{x\to\infty}\left(1+\frac{x}{n}\right)^n \to (1)^\infty$

Hence $\lim_{x\to\infty}\left(1+\frac{x}{n}\right)^n = e^{\lim_{x\to\infty}\left[\left(1+\frac{x}{n}\right)-1\right]}$

$= e^{\lim_{x\to\infty} n\left[\frac{x}{n}\right]}$

$= e^{\lim_{x\to\infty} x}$

$= e^x$

Thus $\lim_{x\to\infty}\left(1+\frac{x}{n}\right)^n = e^x$

2. Let $f(t) = (\sin t)e^{-2t} = \frac{\sin t}{e^{2t}}$

To find $\lim_{x\to\infty}\frac{\sin t}{e^{-2t}}$

Consider $|\sin t| \le |, \forall\, t\, \epsilon /R$

$\Rightarrow -1 \le \sin t \le 1$

$\Rightarrow \frac{-1}{e^{2t}} \le \frac{\sin t}{e^{2t}} \le \frac{1}{e^{2t}}$

Here $\lim_{t\to\infty}\frac{-1}{e^{2t}} = 0$ & $\lim_{t\to\infty}\frac{1}{e^{2t}} = 0$

$\therefore$ By sandwich principle, $\lim_{n\to\infty}\frac{\sin t}{e^{2t}} = 0$

3. $lim_{x\to 8}\left(\frac{x^2-64}{x-8}\right) = lim_{x\to 8}\frac{(x-8)(x+8)}{(x-8)}$

$= lim_{x\to 8}(x+8)$

$= 8 + 8 = 16$

4. $\lim_{x\to\infty} x\tan\left(\frac{1}{x}\right) = \lim_{x\to\infty}\frac{\tan\left(\frac{1}{x}\right)}{\frac{1}{x}} = \lim_{x\to\infty}\frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}}.\frac{1}{\cos\left(\frac{1}{x}\right)} = 1.\frac{x}{1} = 1$

$\Rightarrow \lim_{x\to\infty} x\tan\left(\frac{1}{x}\right) = 1$

5. $lim_{x\to 0}\frac{\sin(x)}{x}\left(\frac{0}{0}\, form\right)$

Using L-Hopital's Rules

$= lim_{x\to 0}\frac{cosx}{1}$

$= \cos 0 = 1$

6. $\lim_{x\to 0}\left(\frac{x\left(1-\frac{\sin 2x}{x}\right)}{x\left(1-\frac{\sin 5x}{x}\right)}\right)$

$$= \lim_{x\to 0}\left(\frac{1-\frac{\sin 2x}{x}}{1-\frac{\sin 5x}{x}}\right)$$

$$= \frac{1-2}{1-5} = 0.25$$

$$\because \lim_{x\to 0}\frac{\sin ax}{x} = a$$

7. $f(x) = \begin{cases} 15, if\ x < 1 \\ Cx, if\ x \geq 1 \end{cases}$

For function $f(x)$ to be continuous

LHS=RHS at $x = 1$

i.e.$lim_{x\to 1^-} f(x) = lim_{x\to 1^+} f(x)$

$lim_{x\to 1^-} 15 = lim_{x\to 1^+} Cx$

C=15

8. $f(x) = 4x - x^2$

$f'(x) = 4 - 2x$

$f'(x) = 0 \Rightarrow 4 - 2x = 0$

$2x = 4 \Rightarrow x = 2$

$f''(x) = -2 < 0$

$\therefore$ function is maximum at $x = 2$

9. Consider the $eq^{\wedge}V = \frac{as}{b+s+\frac{s^2}{9}}$

a=4, b=1, c=9$V = \frac{4S}{1+S+\frac{S^2}{9}}$

Function attains it's maximum or minimum, when v'(s)=0

$\therefore v'(s) = \frac{\left(1+s+\frac{s^2}{9}\right)(4)-4s\left(1+\frac{2s}{9}\right)}{\left(1+s+\frac{s^2}{9}\right)^2}$

Set v'(s) = 0 $\Rightarrow \frac{4+4s+\frac{4s^2}{9}-4s-\frac{8s^2}{9}}{\left(1+s+\frac{s^2}{9}\right)} = 0$

$\Rightarrow 4 - \frac{4s^2}{9} = 0$ $\Rightarrow \left(1 - \frac{s^2}{9}\right) = 0$

$\Rightarrow s^2 = 9$ $\Rightarrow s = \pm 3$

$\therefore$ The positive value at which v(s) is maximum is vs=0

10. For $|\zeta| < |$, we have $\frac{1}{1-\zeta} = \sum_{n=0}^{\infty} \zeta^n = 1 + r + r^2 + r^3 + \ldots$

$\therefore$ For $= 0.3$ i.e. $= \frac{3}{10}$, we have $\frac{1}{1-\frac{3}{10}} = \sum_{n=0}^{\infty}(0.3)^n$

$$\Rightarrow \sum_{n=0}^{\infty}(0.3)^n = \frac{10}{10-3} = \frac{10}{7} = 1.4285$$

11. For geometric series

$a_1, a_2, a_3, a_4, \ldots$

We have, $\frac{a_1}{a_2}=\frac{a_2}{a_3}=\frac{a_3}{a_4}$

Here for R: 1,3,9,27, 81, ……

$\frac{a_1}{a_2} = \frac{1}{3}, \frac{a_2}{a_3} = \frac{3}{9} = \frac{1}{3}, \frac{a_3}{a_4} = \frac{9}{27} = \frac{1}{3}$

$\therefore \frac{a_1}{a_2}=\frac{a_2}{a_3}=\frac{a_3}{a_4}$

$\therefore$ R is Geometric series with common ratio $r = \frac{1}{3}$

S: 4, -8, 16, -32, 64, ……

$\frac{a_1}{a_2} = \frac{4}{-8} = \frac{-1}{2},$

$\frac{a_2}{a_3} = -\frac{8}{16} = -\frac{1}{2}$

$\frac{a_3}{a_4} = \frac{16}{-32} = -\frac{1}{2}$

$\therefore \frac{a_1}{a_2}=\frac{a_2}{a_3}=\frac{a_3}{a_4}$

$\therefore$ S is Geometric series

P: 1,6,11,16,21, 26, ……

$\frac{a_1}{a_2} = \frac{1}{6}$

$\frac{a_2}{a_3} = \frac{6}{11}$

$\therefore \frac{a_1}{a_2} \neq \frac{a_2}{a_3}$

$\therefore$ Not a GP

Q: 9, 6, 3, 0, -3,-6…

$\frac{a_1}{a_2} = \frac{9}{6} = \frac{3}{2},$

$\frac{a_2}{a_3} = \frac{6}{3} = 2$

$\therefore \frac{a_1}{a_2} \neq \frac{a_2}{a_3}$

$\therefore$ Not a GP

12. $S_n = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \ldots + \frac{1}{\sqrt{n}}$, then $\lim_{n\to\infty} S_n = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

Let $a_n = \frac{1}{\sqrt{n}}$

By P-series test, $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent iff p>1

Here $P = \frac{1}{2} < 1 \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^p} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ is diverges

$\therefore S_n$ diverges